A formally verified SSA-based middle-end ⋆
Static Single Assignment meets CompCert

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Abstract. CompCert is a formally verified compiler that generates compact and efficient PowerPC, ARM and x86 code for a large and realistic subset of the C language. However, CompCert foregoes using Static Single Assignment (SSA), an intermediate representation that allows for writing simpler and faster optimizers, and is used by many compilers. In fact, it has remained an open problem to verify formally a SSA-based compiler middle-end. We report on a formally verified, SSA-based, middle-end for CompCert. Our middle-end performs conversion from CompCert intermediate form to SSA form, optimization of SSA programs, including Global Value Numbering, and transforming out of SSA to intermediate form. In addition to provide the first formally verified SSA-based middle-end, we address two problems raised by Leroy [13]: giving a simple and intuitive formal semantics to SSA, and leveraging the global properties of SSA to reason locally about program optimizations.

1 Introduction

Static single assignment Static single assignment (SSA) form [7] is an intermediate representation where variables are statically assigned exactly once. Thanks to the considerable strength of this property, the SSA form simplifies the definition of many optimizations, and improves their efficiency, as well as the quality of their results. It is therefore not surprising that many modern compilers, including GCC and LLVMC [14], rely heavily on SSA form, and that there is a vast body of work on SSA. However, the simplicity of SSA form is deceptive, and designing a correct SSA-based middle-end compiler has been fraught with difficulties. In fact, it has been a significant challenge to design efficient, semantics-preserving, algorithms for converting programs into SSA form, or optimizing SSA programs, or even transforming programs out of SSA form.

Verified Compilers Compiler correctness aims at giving a rigorous proof that a compiler preserves the behavior of programs. After 40 years of a rich history,

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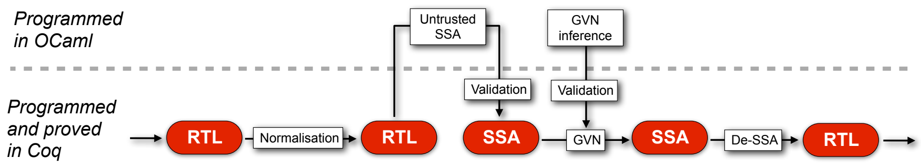


Fig. 1. The SSA Middle-end

the field is entering into a new dimension, with the advent of realistic and mechanically verified compilers. This new generation of compilers was initiated with CompCert [13], a compiler that is programmed and verified in the Coq proof assistant and generates compact and efficient assembly code for a large fragment of the C language. Leroy's CompCert has been rightfully acclaimed as a *tour de force*, but it foregoes relying on an SSA-based middle end. In [13], Leroy reports:

> Since the beginning of CompCert we have been considering using SSA-based intermediate languages, but were held off by two difficulties. First, the dynamic semantics for SSA is not obvious to formalize. Second, the SSA property is global to the code of a whole function and not straightforward to exploit locally within proofs.

add adds: "A typical SSA-based optimization that interests us is global value numbering". However verifying GVN is a significant challenge, and its formal verification has remained beyond current state-of-the-art in certified compilers.

Static Single Assignment meets verified compilers The thesis of our work is that a compiler can be realistic, verified and still rely on a SSA form. To support our thesis, we provide the first verified SSA-based middle-end. Rather than programming and proving a verified compiler from scratch, we have programmed and verified a SSA-based middle-end compiler that can be plugged into CompCert at the level of RTL. Fig. 1 describes the overall architecture. Our middle-end performs four phases: (i) normalization of RTL program; (ii) transformation from RTL form into SSA form; (iii) optimization of programs in SSA form, including Global Value Numbering (GVN) [1]; (iv) transformation of programs from SSA form to RTL form; and relies on CompCert for the transformation from C to RTL programs prior to SSA conversion, and from RTL programs to assembly code after conversion out of SSA—our point is to program a realistic and verified SSA-based middle-end, rather than to demonstrate that SSA-based optimizations dramatically improve the efficiency of generated code.

We validate our compiler middle-end with a mix of techniques directly inherited from CompCert. We resort to translation validation [19, 18]—increasingly favored by CompCert [24, 25]—for converting programs into SSA form and for GVN. Specifically, we program in Coq verified checkers that validate *a posteriori* results of untrusted computations, and we implement in OCaml efficient algorithms for these computations; we rely on Cytron *et al* algorithm [7] for computing minimal SSA form, and on Alpern *et al* iteration strategy [1] for computing a numbering in GVN. In contrast, the normalization of the RTL program, and the conversion out of SSA are directly programmed and proved in Coq. In addition, our work addresses the two issues raised by Leroy [13]. First, we give a simple and intuitive operational semantics for SSA; the semantics follows the informal description given in [7], and does not require any artificial state instrumentation. Second, we define on SSA programs two global properties, called strictness and equational form, allowing to conclude reasonably directly that the substitutions performed by GVN and other optimizations are sound.

Summarizing, our work provides the first verified SSA-based middle-end, the first formal proof of an SSA-based optimization, as well as an intuitive semantics for SSA. It thus serves as a good starting point for further studies of verified and realistic SSA-based compilers.

Contents The paper is organized as follows: Section 2 provides a brief primer on SSA and CompCert. Section 3 defines the SSA language used by our middle-end. Conversion to and out of SSA forms are presented in Section 4 and 5 respectively. Section 6 presents SSA-based optimizations. We conclude with experimental results in Section 7 and related work in Section 8.

Throughout the paper, we use Coq syntax for our definitions and results. Statements occasionally involve some notions that are not introduced formally. In such cases, names are generally chosen to be self-explanatory (for instance, `not_wrong_program`); in other cases, we forego giving precise definitions as they are needed to understand the paper (for instance, the types `chunk` and `addressing` are unspecified in the definition of state). Our formalization makes an extensive use of inductive definitions, which are introduced in Coq using the keyword **`Inductive`**. Inductive definitions are used both for introducing new datatypes, e.g. the type of RTL instructions in Fig. 4, and for introducing inductive relations, e.g. the operational semantics of RTL instructions in Fig. 4. In the latter case, the declarations are written according to the pattern `Inductive R : A →B →Prop := | Rule1: ∀ a b, ... →R a b | Rule2:....`

2 Background

Static Single Assignment form is an intermediate representation in which variables are statically assigned exactly once, thus making explicit in the program syntax the link between the program point where a variable is defined and read.

Converting into SSA form is easy for straighline code: one simply tags each variable definition with an index, and each variable use with the index corresponding to the last definition of this variable. For example, $[x := 1; y := x + 1; x := y - 1; y := x]$ is transformed into $[x_0 := 1; y_0 := x_0 + 1; x_1 := y_0 - 1; y_1 := x_1]$. The transformation is semantics-preserving, in the sense that the final values of x and y in the first snippet coincide with the final values of x_1 and y_1 in the second snippet. On the other hand, one cannot transform arbitrary programs

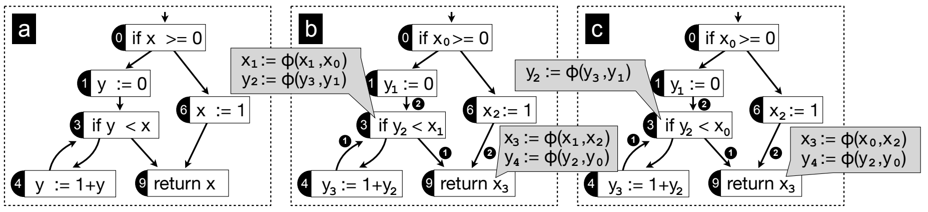


Fig. 2. Example programs. Programs b), c) are SSA forms of a). Programs b) is in naive form and c) in minimal form.

into semantically equivalent programs in SSA form solely by tagging variables: one must insert *ϕ-functions* to handle branching statements. Fig. 2 shows a program a), and a program b) that corresponds to a SSA form of a). In program a), the value of variable x read at node 9 either comes from the definition of x at entry or at node 6. In program b), these two definitions of x are renamed into the unique definition of x_0 and x_2 and merged together by the ϕ-function of x_3 at entry of node 9. The precise meaning of a ϕ-block depends on the numbering convention of the predecessor nodes of each junction point. In Fig. 2 b) we make explicit this numbering by labelling the CFG edges. For example, node 3 is the first predecessor of point 9 and node 6 is the second one. The semantics of ϕ-functions is given in the seminal paper by Cytron *et al* [7]:

"If control reaches node j from its kth predecessor, then the run-time support remembers k while executing the ϕ-functions in j. The value of $\phi(x_1, x_2, \ldots)$ is just the value of the kth operand. Each execution of a ϕ-function uses only one of the operands, but which one depends on the flow of control just before entering j."

There may be several SSA forms for a single control-flow graph program; Fig. 2 b) and c) gives alternative SSA forms for program a). As the number of ϕ-functions directly impacts the quality of the subsequent optimizations—as well as the size of the SSA form—it is important that SSA generators for real compilers produce a SSA form with a minimal number of ϕ-functions. Implementations of minimal SSA generally rely on the notion of *dominance frontier* to choose where to insert ϕ-functions. A node i in a CFG dominates another node j if every path from then entry of the CFG to j contains i. The dominance is said to be strict if additionally $i \neq j$. A tree can encode the dominance relation between the nodes of the CFG. For a node i of a CFG, the *dominance frontier* $DF(i)$ of i is defined as the set of nodes j such that i dominates at least one predecessor of j in the CFG but does not strictly dominates j itself. The notion is extended to a set of nodes S with $DF(S) = \bigcup_{i \in S} DF(i)$. The *iterated dominance frontier* $DF^+(S)$ of a set of nodes S is $\lim_{i\to\infty} DF^i(S)$, where $DF^1(S) = DF(S)$ and $DF^{i+1}(S) = DF(S \cup DF^i(S))$. Formally, a program is in *minimal-SSA* form when a ϕ-function of an instance x_i of an original variable x appears in a junction point j iff j belongs to the iterated dominance frontier of the set of definition nodes of x in the original program. For instance, program c) in Fig. 2 is in minimal-SSA

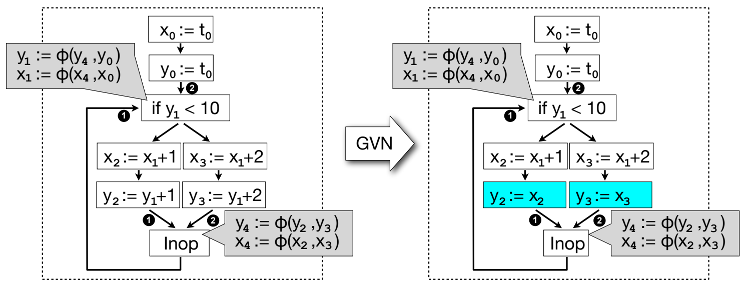


Fig. 3. Common sub-expression elimination (CSE) using GVN

form. However, one can achieve more compact SSA forms by observing that, at any junction point, dead variables need not be defined by a ϕ-function. The intuition is captured by the notion of *pruned-SSA* form: a program is in *pruned-SSA* form if it is in minimal-SSA form and for each ϕ-function of an instance x_i of an original variable x at a junction point j, x is live at j in the original program (there is a path from j to a use of x that does not redefine x).

SSA-based optimizations The SSA form simplifies the definition of many common optimizations; for instance, copy propagation algorithms can just walk through a SSA program, identify statements of the form $x := y$, and replace every use of x by y. Furthermore, several optimizations are naturally formulated on SSA. One typical SSA-based optimization is *Global Value Numbering* (GVN) [1], which assigns to variables an identifying number such that variables with the same number will hold equal values at execution time. The effectiveness of GVN lies in its ability to compute efficiently numberings that identify as many variables as possible. Advanced algorithms [1, 5] allow to compute efficiently such numberings. We briefly explain one such numbering in Section 6.

Fig. 3 illustrates how GVN can be used to eliminate redundant computation. The left program is the original code; in this program, for each i, x_i and y_i are assigned the same value number. Hence, the evaluation of y_1+1 (resp. y_1+2) is a redundant computation when assigning y_2 (resp. y_3), and one can transform the program into the semantically equivalent one shown on the right of the figure. The strength of the analysis lies in its ability to reason about ϕ-functions, which allows it to infer the equality $x_2 = y_2$. This is only possible because numbering is global to the whole program; in fact, any block-local analysis would fail to discover the equality $x_2 = y_2$.

CompCert is a realistic formally verified compiler that generates PowerPC, ARM or x86 code from source programs written in a large subset of C. CompCert formalizes the operational semantics of dozen intermediate languages, and proves for each phase a semantics preservation theorem. Preservation theorems are expressed in terms of program behaviors, i.e. finite or infinite traces of external function calls (a.k.a. events) that are performed during the execution of the program, and claim that individual compilation phases preserve behaviors. A consequence of the theorems is that for any C program `p` that does not go wrong, and target program `tp` output by the successful compilation of `p` by the compiler `compcert_compiler`, the set of behaviors of `p` contains all behaviors of the target program `tp`. The formal theorem is:

```
Theorem compcert_compiler_correct: ∀ (p: C.program) (tp: Asm.program),
  (not_wrong_program p ∧ compcert_compiler p = OK tp) →
  (∀ beh, exec_asm_program tp beh → exec_C_program p beh).
```

This paper focuses on the CompCert middle-end where most of the existing optimisations are performed (currently: constant propagation, removal of redundant cast, tail call detection, local value numbering and a register allocation phase that includes copy propagation). These operate on a Register Transfer Language (RTL), whose syntax and semantics is given in Fig. 4. A RTL program is defined as a set of global variables, a set of functions, and an entry node. Functions are modelled as records that include a function signature `fn_sig`, a CFG `fn_code` of instructions over pseudo-registers. The CFG is not a basic-block graph: it partially maps each CFG node to a single instruction, and we stick to this important design choice of CompCert. As explained by Knoop *et al* [10], it allows for simpler implementations of code manipulations and simplifies correctness proofs of analyses or transformations, without impacting too much their efficiency.

The RTL instruction set includes arithmetic operations (`Iop`), memory loads (`Iload`) and stores (`Istore`), function calls (`Icall`), conditional (`Icond`) and unconditional jumps (`Inop`), and a return statement (`Ireturn`)— for brevity, we do not discuss here jumptables and other kinds of function calls: call to a function pointer stored in a register, tail calls, and built-in functions. All instructions take as last argument a node `pc` denoting the next instruction to be executed; additionally, all instructions but `Inop` take as arguments pseudo-registers of type `reg`, memory chunks, and addressing modes.

The type of states is defined as the tagged union of regular states, call states and return states (Fig. 4). We focus on regular states, as we only expose here the intra-procedural part of the language. A regular semantic state (`State`) is a tuple that contains a call stack (representing the current pending function calls), the current function description and stack pointer (to the stack data block, a part of the global memory where variables dereferenced in the C source program reside), the current program point, the registers state (a mapping of local variables to values) and the global memory. The semantics also includes a global environment mapping function names and global variables to memory addresses; it is never modified during a program execution, and thus ommitted in our presentation.

The operational behavior of programs is modelled by the relation `step` between two semantic states (see Fig. 4), and a trace of events; all instructions except function calls do not emit any event, hence the transitions that they induced are tagged by the empty event trace ϵ. We briefly comment on the rules: (`Inop pc′`) branches to the next program point `pc′`. (`Iop op args res pc′`) performs the arithmetic operation `op` over the values of registers `args` (written `rs##args`), stores the result in `res` (written `rs#res ← v`), and branches to `pc′`. The instruction (`Iload chk addr args res pc′`) loads a `chk` memory quantity

```
Inductive instr  :=
  | Inop (pc: node)
  | Iop (op: operation) (args: list reg) (res: reg) (pc: node)
  | Iload (chk:chunk) (addr:addressing) (args: list reg) (res: reg) (pc: node)
  | Istore (chk:chunk) (addr:addressing) (args:list reg) (src: reg) (pc: node)
  | Icall (sig: signature) (fn:ident) (args: list reg) (res: reg) (pc: node)
  | Icond (cond: condition) (args: list reg) (ifso ifnot: node)
  | Ireturn (or: option reg).

Inductive state :=
 | State (stack: list stackframe) (* call stack *)
         (f: function)            (* current function *)
         (sp: val)                (* stack pointer *)
         (pc: node)               (* current program point *)
         (rs: regset)             (* register state *)
         (m: mem)                 (* memory state *)
 | Callstate (stack: list stackframe) (f: fundef) (args: list val) (m: mem)
 | Returnstate (stack: list stackframe) (v: val) (m: mem).

Inductive step: state → trace → state → Prop :=
 | ex_Inop: ∀ s f sp pc rs m pc',
     fn_code f pc = Some(Inop pc') →
     step (State s f sp pc rs m) ϵ (State s f sp pc' rs m)
 | ex_Iop: ∀ s f sp pc rs m pc' op args res v,
     fn_code f pc = Some(Iop op args res pc') →
     eval_operation sp op (rs##args) m = Some v →
     step (State s f sp pc rs m) ϵ (State s f sp pc' (rs#res←v) m)
 | ex_Iload: ∀ s f sp pc rs m pc' chk addr args res a v,
     fn_code f pc = Some(Iload chk addr args res pc') →
     eval_addressing sp addr (rs##args) = Some a →
     Mem.loadv chk m a = Some v →
     step (State s f sp pc rs m) ϵ (State s f sp pc' (rs#res←v) m)
```

Fig. 4. Syntax and semantics of RTL (excerpt)

from the address determined by the addressing mode `addr` and the values of the `args` registers, stores the quantity just read into `res`, and branches to `pc'`.

3 The SSA language

We describe the syntax and operational semantics of the language SSA that provides the SSA form of RTL programs. We equip the notion of SSA program with a *well-formedness* predicate capturing essential properties of SSA forms.

SSA programs Our definition of SSA program distinguishes between RTL-like instructions and ϕ-functions; the distinction avoids the need for unwieldy mappings between program points when converting to SSA, and allows for a smooth integration in CompCert. Fig. 5 introduces the syntax of SSA. SSA functions operate on indexed registers of type `SSA.reg` $=$ `RTL.reg` $*$ `idx`, and include an additional field `fn_phicode` mapping junction points to ϕ-blocks. The latter are modelled as lists of ϕ-functions of the form (`Iphi args res`), where `res` is an indexed register, and `args` a list of indexed registers.

Next, we define structural constraints that allow giving an intuitive semantics to SSA programs. First, we require that the domain of the function fn_phicode be the set of junction points. Second, we require that all ϕ-functions in a ϕ-block have the same number of arguments as the number of predecessors of that block. Third, we require that all predecessors of a junction point be (Inop pc) instructions. This is a mild constraint, that can be ensured systematically on RTL programs through normalization, and that will carry over to their SSA forms. Fig. 6 shows the RTL program from Fig. 2 after normalization.

Finally, we consider two essential properties of SSA forms: unique definitions and strictness. The unique definitions property states that each register is uniquely defined, whereas the strictness property states that each variable use is dominated by the (unique) definition of that variable. While the two properties are closely related, none implies the other; the program $[y_0 := x_0; x_0 := 1]$ satisfies the unique definitions property but is not in strict form whereas the program $[x_0 := 1; x_0 := 2; y_0 := x_0]$ is strict but does not satisfy the unique definitions property. To formalize these properties, one first defines the type of paths in a CFG, and predicates dom and sdom for dominance and strict dominance. Then, one must define the two predicates def, use of type SSA.function $\rightarrow$ SSA.reg $\rightarrow$ node $\rightarrow$ Prop such that proposition def f x pc (respectively use f x pc) holds iff the register x is defined (resp. used) at node pc in the (RTL-like or ϕ-) code of the function f. The definition of use is complex because variables may be used in ϕ-functions: the widely adopted convention is to view ϕ-functions as lazily evaluated, their ith argument thus being used at the ith predecessor of the instruction. For example, in the SSA program of Fig. 6, variable x$_2$ is defined at node 6 and used at node 8, the 2nd predecessor of the junction point 9 where x$_2$ appears as 2nd argument of the ϕ-function. A use in the regular code is more straightforward: a variable is used by an instruction if it appears on its right-hand side. Using def and use, one can then state the unique definition and strictness properties, and well-formedness. Formally, we say that a SSA function is well-formed if it satisfies the following predicate:

```
Record wf_ssa_function (f:SSA.function) : Prop := {
  fn_ssa:           unique_def f;
  fn_strict:        ∀ x u d, use f x u → def f x d → dom f d u;
  fn_wf_block:      block_nb_args f;
  fn_block_at_jp:   ∀ jp, join_point jp f ↔ fn_phicode f jp ≠ None;
  fn_normalized:    ∀ jp pc, join_point jp f → In jp (succs f pc) →
                              fn_code f pc = Some (Inop jp);}.
```

where predicates unique_def and block_nb_args respectively capture that a function satisfies the unique definitions property and the structural constraint about arguments. In the sequel, we show that conversion to SSA yields well-formed programs. Besides, our SSA-based optimizations will assume that the input SSA programs are well-formed; in turn, we prove for each of them that output programs are well-formed.

Semantics The notion of SSA state is similar to the notion of RTL state, except that the type of registers and current function are modified into SSA.reg and SSA.function respectively. The small-step operational semantics is defined on

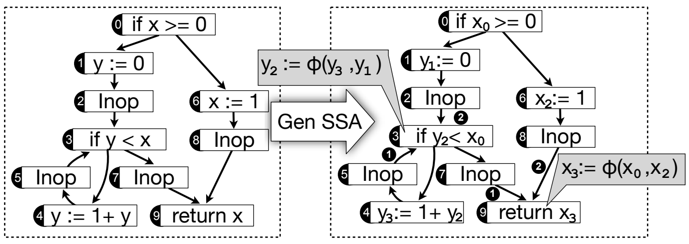
```
Inductive instr := ...                   Record function := {
                                          fn_sig: signature;        signature
Inductive phiinstr :=                     fn_params: list SSA.reg;  parameters
 | Iphi (args: list SSA.reg)              fn_stacksize: Z;    activation record size
        (res: SSA.reg).                   fn_code: code;            code graph
                                          fn_phicode: phicode;      ϕ-blocks graph
Definition phiblock:= list phiinstr.      fn_entrypoint: node}.     entry node

Inductive step: SSA.state → trace → SSA.state → Prop :=
  | ex_Inop_njp: ∀ s f sp pc rs m pc',
     fn_code f pc = Some(Inop pc') →
     ¬ join_point pc' f →
     step (State s f sp pc rs m) ϵ (State s f sp pc' rs m)
  | ex_Inop_jp: ∀ s f sp pc rs m pc' phib k,
     fn_code f pc = Some(Inop pc') →
     join_point pc' f →
     fn_phicode f pc' = Some phib →
     index_pred f pc pc' = Some k →
     step (State s f sp pc rs m) ϵ (State s f sp pc' (phistore k rs phib) m)

Fixpoint phistore k rs phib : nat → SSA.regset → phiblock → SSA.regset :=
  match phib with
    | nil => rs
    | (Iphi args res)::phib =>
      match nth_error args k with
        | None => rs
        | Some arg => (phistore k rs phib)#res ← (rs#arg)
      end end.
```

Fig. 5. Syntax and semantics of SSA (excerpt)

SSA programs that satisfy the structural constraints introduced in the previous paragraph. Formally, we define SSA.step as a relation between pairs of (SSA) states and a trace of events. The definition follows the one of RTL.step, except for instructions of the form (Inop pc′), where one distinguishes whether pc′ is a junction point or not. In the latter case, the semantics coincide with the RTL semantics, i.e. the program point is updated in the semantic state. If on the contrary pc′ is a junction point, then one executes the ϕ-block attached to pc′ before the control flows to pc′. Executing ϕ-blocks on the way to pc′ avoids the need to instrument the semantics of SSA with the predecessor program point, and crisply captures the intuitive meaning given to ϕ-blocks by Cytron *et al* (see Section 2). Note in particular that the normalization ensures the predecessor of a junction point is an Inop instruction. This greatly simplifies the definition of the semantics, and subsequently the proofs about SSA programs.

Following conventional practice, ϕ-blocks are given a parallel (big-step) semantics. This is formally embedded in the rule for phistore (Fig. 5). When reaching a join point pc′ from its kth predecessor, we update the register set rs for each register res assigned in the ϕ-block phib with the value of register arg in rs (written rs#arg), where arg is the kth operand in the ϕ-function of res (written nth_error args k = Some arg). With the same notations, phistore satisfies, on well-formed SSA functions, a *parallel assignment* property:

```
 ∀ arg res, In (Iphi args res) phib →
    nth_error args k = Some arg → (phistore k rs phib)#res = rs#arg
```



pc	$(\Gamma$ pc x)	$(\Gamma$ pc y)	(live pc)
0	0	0	{x}
1	0	0	{x}
2	0	1	{x, y}
3	0	2	{x, y}
4	0	2	{x, y}
5	0	3	{x, y}
6	0	0	{ }
7	0	2	{x}
8	2	0	{x}
9	3	0	{x}

Fig. 6. A RTL program, its pruned SSA form and the corresponding type information

4 Translation validation of SSA generation

Modern compilers typically follow the algorithm by Cytron *et al* [7] to generate a minimal SSA form of programs in almost linear time w.r.t. the size of the program. The algorithm proceeds in four steps: (i) it computes the dominator tree of the CFG using the Lengauer and Tarjan algorithm [12]; (ii) it builds the dominance frontier using a bottom-up traversal of the dominator tree; (iii) for each variable, it places ϕ-functions using iterated dominance frontier; (iv) at last, it uses a top-down traversal of the dominator tree to rename each def and use of RTL variables with correct indexes. Programming efficiently the algorithm in Coq and proving formally its correctness is a significant challenge—even verifying formally Step (i) requires to formalize a substantial amount of graph theory. Instead, we provide a new validation algorithm that checks in linear time that a SSA program is a correct SSA form of an input RTL program. The algorithm is complete w.r.t. minimal SSA form, and can be enhanced by a liveness analysis to handle pruned and semi-pruned SSA forms. In order to be used in a certified compiler chain, we also show that our validator preserves behaviors.

Translation validation of SSA conversion is performed in two passes. The first pass performs a structural verification on programs: given a RTL function `f` and a SSA function `tf`, it verifies that `tf` satisfies all clauses of well-formedness except strictness, and that the code of `f` can be recovered from its SSA form `tf` simply by erasing ϕ-blocks and variable indices—the latter property is captured formally by the proposition `structural_spec f tf`. The second pass relies on a type system to ensure strictness and semantics-preservation. Overall the pseudo-code of the validator is:

```
let SSA_validator (f: RTL.function) (tf: SSA.function): bool :=
  if    (check_blocks_are_wf tf)          (* ensures block_are_wf tf *)
     && (check_blocks_are_at_jp tf)       (* ensures block_at_jp tf *)
     && (check_normalized tf)             (* ensures normalization *)
     && (check_unique_def tf)             (* ensures unique_def tf *)
     && (check_structural_spec f tf)      (* ensures structural_spec f tf *)
  then (is_well_typed f tf) else false
```

where `is_well_typed f tf` is the predicate stating that the function is well-typed w.r.t. our type system for SSA form.

```
Definition use_ok (uses:list SSA.reg)(γ:ltype):= ∀ r i, In (r,i) uses→ γ r=i.

Inductive wt_instr: ltype → SSA.instr → ltype → Prop :=
| wt_Inop: ∀ γ s, {γ} Inop s {γ}
| wt_Istore: ∀ γ chk addr args s src,
      use_ok (src::args) γ → {γ} Istore chk addr args src s {γ}
| wt_Icond: ∀ γ cond args s1 s2,
      use_ok args γ → {γ} Icond cond args s1 s2 {γ}
| wt_Ireturn_some: ∀ γ r, use_ok [r] γ → {γ} Ireturn (Some r) {γ}
| wt_Ireturn_none: ∀ γ, {γ} Ireturn None {γ}
| wt_Iop: ∀ γ op args s r i,
      use_ok args γ  → i ≠ dft → {γ} Iop op args (r,i) s {γ[r ← i]}
| wt_Iload: ∀ γ chk addr args s r i,
      use_ok args γ → i ≠ dft → {γ} Iload chk addr args (r,i) s {γ[r ← i]}
| wt_Icall: ∀ γ sig args s id r i,
      use_ok args γ → i ≠ dft → {γ} Icall sig id args (r,i) s {γ[r ← i]}
```

Fig. 7. Typing rules for instructions

Type system The basic idea of our type system is to track for each variable its *last* definition; this is achieved by assigning to all program points a local typing, i.e., an element of `ltype` = `RTL.reg` → `idx`; we let γ range over local typings. Then, the global typing of an SSA function `tf` is an element of `gtype` = `node` → `RTL.reg` → `idx`; we let Γ range over global typings. The type system is structured in three layers. The lowest layer checks that RTL-like instructions make a correct use of variables. The middle layer checks that CFG edges are well-typed. Finally, the third layer of the type system defines the notion of well-typed function. Throughout this section, we use Fig. 6 as a running example (an RTL program, its pruned SSA form and its type mapping).

Liveness As explained in Section 2, liveness information can be used to minimize the number of ϕ-functions in a SSA program; specifically, ϕ-blocks only need to assign live variables (in Fig. 6, the variable y is live at node 3, and x is live at node 9). Hence, our type system is parametrized by a function `live` that models a liveness analysis. Formally, we require that the `live` function satisfy two properties (for a function `f`, their conjunction is denoted by (`wf_live f live`)): (i) if a variable is used at a program point, then it should be live at this point and (ii) a variable that is live at a given program point is, at the predecessor point, either live or assigned.

Our type system is able to handle different SSA forms through appropriate instantiations of `live`. Our formalization provides support for minimal SSA and pruned SSA forms, respectively by defining `live` respectively as the trivial overapproximation (for each point, it is the set of all the RTL variables), and the result of a standard liveness analysis. One could also support semi-pruned forms, by instantiating `live` as the result of the block-local liveness analysis of [4].

The type system for instructions checks that RTL-like instructions make of correct use of variables, and that they do not redefine parameters; its formal definition is given in Fig. 7. Judgments are of the form $\{\gamma\}$ `ins` $\{\gamma'\}$; intuitively, the judgment is valid if each variable x is used in ins with the index (γ x), and γ' maps each variable to its last definition after execution of ins. The typing rules are formalized as an inductive relation wt_instr; we briefly comment on some rules. Several rules correspond to instructions that do not define variables, so the input and output local typings are equal. For such rules, one simply checks that the instruction makes a correct use of the variables (through use_ok). The typing rule for (Inop pc) states that for every local typing γ, (Inop pc) makes a correct use of variables. The typing rule for Icond checks that the variables used in the guard are consistent with the local typing input (in Fig. 6, the uses of x_0 and y_2 at node 3 are consistent with the intput local typing: (Γ 3 x) $= 0$ and (Γ 3 y) $= 2$). In the case of the instruction Iop, which defines the variable (r, i), the output local typing is $\gamma[\mathtt{r} \leftarrow \mathtt{i}]$, i.e. the input local typing updated for the initial variable r. From this program node onwards, the new version for r is the one indexed with i, and this is the one that should be used later on, until another version for r is defined (in Fig. 6, the definition of x_2 at node 6 makes the local typing change for variable x between nodes 6 and 8). Note that each time a variable is defined, we demand its index to be different from the index dft assigned to parameters at the onset of the program (in the example, the default index is 0). This prevents that a parameter is redefined during execution, which would violate the unique definition property.

Typing rules for edges and functions The typing rules for edges ensure that ϕ-blocks make a correct use of definitions w.r.t. a global typing Γ. There are two rules—modelled by the clauses of the inductive relation wt_edge in Fig. 8. The first rule considers the case where the edge does not end in a junction point; in this case, typing the edge is equivalent to typing the corresponding instruction. The second rule considers the case where the edge ends in a junction point: the typing rule checks the ϕ-block attached to it—structural constraints impose that the instruction is an Inop, so we do not need to type-check the instruction. Hypothesis USES ensures the ϕ-arguments args passed to ϕ-functions are consistent w.r.t. all incoming local typings: its kth argument should be the version of the initial variable brought by the kth predecessor of the join point (we omit the formal definition of phiuse_ok). Hypothesis ASSIG ensures the ϕ-block is compatible with the output local typing; Hypothesis NASSIG ensures that variables not assigned in the ϕ-block are either dead, or the incoming indices are the same. In Fig. 6, the ϕ-function for x makes correct uses of it because its first argument x_0 matches (Γ 7 x) $= 0$ and x_2 matches (Γ 8 x) $= 2$. The local typing at node 9 takes into account the definition of x_3 in the block by setting (Γ 9 x) to 3. Moreover, no ϕ-function is required for y at node 9 since y $\notin$ (live 9), and no ϕ-function is required for x at node 3, since (Γ 2 x) $=$ (Γ 5 x).

Finally, a function is well-typed w.r.t. global typing Γ if the local typing induced by Γ at the entry node fn_entrypoint is consistent with the parameters, and all edges and return instructions are well-typed.[4]

[4] Return instructions do not correspond to any edge.

```
Inductive wt_edge (f:SSA.function)(Γ:gtype)(live:Regset.t):node→ node → Prop:=
| wt_edge_not_jp: ∀ i j ins
(NOTJP : fn_code f i = Some ins ∧ fn_phicode f j = None)
(WTI :    {Γ i} ins {Γ j}),
(wt_edge f Γ live i j)

| wt_edge_jp: ∀ i j ins block
(JP:  fn_code f i = Some ins ∧ fn_phicode f j = Some block)
(USES:∀ args r k, In (Iphi args (r,k)) block → phiuse_ok r args (preds f j) Γ)
(ASSIG: ∀ r k, assigned (r,k) block → r ∈ live ∧ (Γ j r) = k ∧ k ≠ dft)
(NASSIG: ∀ r, (∀ k, ¬ (assigned (r,k) block)) → (Γ i r = Γ j r) ∨ r ∉ live),
(wt_edge f Γ live i j).

Definition wt_function (f:SSA.function)(Γ:gtype)(live:node→ Regset.t):Prop:=
(∀ i j, is_edge f i j → wt_edge f Γ (live j) i j)
∧ (∀ i or, fn_code f i = Some (Ireturn or) → {Γ i} Ireturn or {Γ i})
∧ (∀ p, In p (fn_params f) → ∃ r, p = (r, Γ (fn_entrypoint f) r)).
```

Fig. 8. Typing rules for edges and functions

Implementation For the sake of clarity, we have described a non-executable type checker which assumes that structural constraints are satisfied. The Coq implementation of the type system is in fact a bit more complex. In particular, it performs type inference rather than type checking; for efficiency reasons, the algorithm performs a single, linear scan of the program, and checks the list of arguments of ϕ-functions only once per junction point, rather than once per incoming edge for a given join point. On the benchmarks given in Section 7, our efficient implementation is ten times faster than a naive type checker derived from the non-executable type system.

Properties of the validator

Strictness All SSA programs accepted by the type system are strict. It follows that only well-formed SSA functions will be accepted by the validator.

```
Theorem wt_strict: ∀ f tf Γ live,
wf_live f live → wt_function tf Γ live →
∀ (xi : SSA.reg) (u d : node), use tf xi u → def tf xi d → dom tf d u.
```

The proof of `wt_strict` relies on two auxiliary lemmas about local typings in well-typed functions. The first lemma states that if a variable $(\mathtt{x},\mathtt{i})$ is used at node `pc`, then it must be that $(\Gamma\ \mathtt{pc}\ \mathtt{x} = \mathtt{i})$. The second lemma states that whenever $(\Gamma\ \mathtt{pc}\ \mathtt{x} = \mathtt{i})$, the definition point of variable $(\mathtt{x},\mathtt{i})$ dominates `pc`.

Soundness The validator is sound in the sense that if it accepts a RTL program `f` and an SSA form `tf`, then all behaviors of `tf` are also behaviors of `f`. Since CompCert already shows the general result that a lock-step forward simulation implies preservation of behaviors, it is sufficient to exhibit such a simulation:

```
Theorem validator_correct : ∀ (prog:RTL.program) (tprog:SSA.program),
SSA_validator prog tprog = true →
 ∀ s1 t s2, RTL.step s1 t s2 →
   ∀ s1', s1 ≃ s1' → ∃ s2', SSA.step s1' t s2' ∧ s2 ≃ s2'.
```

where the binary relation $\simeq$ between semantic states of RTL and SSA carries the invariants needed for proving behavior preservation. For instance, two regular states are related by $\simeq$ if their memory states, stack pointers, and program counters are equal, their function descriptions are suitably related, e.g. by `structural_spec`, and their register states `rs` and `rs'` agree, i.e. satisfy (`agree` (Γ `pc`) `rs` `rs'` (`live` `pc`)), where

```
Definition agree (γ:ltype) (rs:RTL.regset) (rs':SSA.regset) (live:Regset.t):=
  ∀ r, r ∈ live → rs#r  = rs'#(r, γ r).
```

Agreement is at the heart of the proof. It captures the semantics of local typings by making explicit how, at a given program point, variables of `f` should be interpreted in terms of the new variables in `tf`. The definition of $\simeq$ is completed by defining equivalence of stackframes; this relation basically lifts to the callstack all the invariants enforced by $\simeq$ (see [6] for the formal definition of $\simeq$).

Completeness An essential property of our type system is that it accepts all the SSA programs that are output by the algorithm by Cytron *et al* [7]. The idea of the proof is as follows (provided in [6]). First, one defines for each RTL normalized program `f` a global typing Γ. Second, we show that all instructions of the program `tf` output by our implementation are typable. Then, we show that all edges are typable if we omit the constraints about correct use; the proof relies crucially on the fixpoint characterization of the iterated dominance frontier, as given in work of Cytron *et al* [7]. Finally, one shows that all constraints about correct use are satisfied, and hence the program `tf` is typable with Γ.

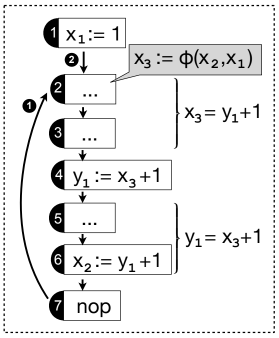
5 Conversion out of SSA

We have programed and verified a simple de-SSA algorithm that transforms SSA programs into RTL programs—so that they can be further processed by CompCert back end. The idea is to substitute each ϕ-function with one variable copy at each predecessor of junction points. Thanks to the single-instruction graph of RTL, replacing ϕ-functions with copies ensures soundness of the transformation, since critical edges are automatically splitted by code insertion—a critical edge is an edge whose entry has several successors and exit has several predecessors (see [4]). Pleasingly, the representation of programs inherited from CompCert deflates the penalty cost of splitting edges—on the contrary, algorithms that operate on *basic-block graphs* carefully avoid edge splitting, at the cost of making de-SSA algorithms significantly more complex. On the negative side, our current implementation of de-SSA fails on SSA programs with non-parallel ϕ-blocks, i.e. in which some variable is both used and defined. Minor future work includes making de-SSA total, reusing the formalization of the parallel moves algorithm [20]—which transforms a set of parallel moves into an equivalent sequence of elementary moves (using additional temporaries), and that is already used in CompCert. Concerning the correctness of the transformation, we proceed by giving a forward simulation between the SSA program and the RTL program after de-SSA. The simulation requires the RTL program to perform several steps to simulate a (big-step) execution of a ϕ-block by the initial SSA program.

6 Validation of SSA-based optimizations

In this section, we introduce the *equation lemma* that supports the view of programs in SSA form as *systems of equations*. We then illustrate how to reason about a simple SSA-based optimization, namely copy propagation. Finally, we formalize and prove correct a GVN optimization.

Equation lemma The SSA representation provides an intuitive reading of programs: one can view the unique definition of a variable as an equation, and by extension one can view SSA programs as systems of equations. For instance, the definitions of $\mathtt{x}_3$ and $\mathtt{y}_1$ respectively induce the two equations $x_3 = y_1 + 1$ and $y_1 = x_3 + 1$. There is however a pitfall: the two equations entail $x_3 = x_3 + 2$, and thus are inconsistent. In fact, equations are only valid at program nodes dominated by the definition that induce them, as captured formally by the *equation-lemma* of SSA:



```
Lemma equation_lemma : ∀ prog d op args x succ f m rs sp pc s,
    wf_ssa_program prog →
      reachable prog (State s f sp pc rs m) →
      fn_code f d  = Some (Iop op args x succ) →
      sdom f d pc →
      eval_operation sp op (rs##args) m = Some (rs#x).
```

where `reachable` is a predicate that defines reachable states. In practice, it is often convenient to rely on a corollary that proves the validity of the defining equation of `x` at program points where `x` is used – thus avoiding reasoning on the dominance relation. The formal statement of the corollary is obtained by replacing the hypothesis `sdom f d pc` by the hypothesis `use f x pc`; the proof of the corollary intensively uses the strictness property of well-formed SSA programs.

We conclude with a succinct account of applying the corollary to prove the soundness of copy propagation (CP)—recall that CP will search for copies `x` := `y` and replace every use of `x` by a use of `y`. Suppose `pc` is a program point where such a replacement has been done. Every time `pc` is reached during the program execution, we are able to derive, using the corollary, that `rs`#`y` = `rs`#`x`, where `rs` is the current register state because (i) `y` is the right hand side of the definition of `x` and (ii) `pc` was a use point of `x` in the initial program. On non-SSA forms, the reasoning is more involved since one has to prove that the reaching definition for `x` is unique at `pc`, and that no redefinition of `y` can occur in between.

Global Value Numbering Our implementation of GVN is made of two components. The first one is an efficient but untrusted analysis, written in OCaml, for computing numberings of SSA programs. From an abstract interpretation point of view, the analysis—which follows [1]—computes a fixpoint in the abstract domain of congruence partitions, where partitions are modelled as mappings $\mathcal{N}$: `reg` → `reg` that map a register to the canonical register of its equivalence

```
Inductive  ≡^𝒩  : reg → reg → Prop :=
| GVN_refl : ∀ x,  ≡^𝒩  x x
| GVN_Iop : ∀ x y pc1 pc2 op args1 args2 pc1' pc2'
    fn_code f pc1 = Some(Iop op args1 x pc1') → same_number 𝒩 args1 args2 →
    fn_code f pc2 = Some(Iop op args2 y pc2') →  ≡^𝒩  x y
| GVN_Phi : ∀ x y pc args_x args_y
    fn_phicode f pc = Some phib → same_number 𝒩 args_x args_y →
    (Iphi args_x x) ∈ phib → (Iphi args_y y) ∈ phib →  ≡^𝒩  x y.

Definition GVN_spec (𝒩:reg → reg) : Prop :=
(∀ x y, 𝒩 x = 𝒩 y → param f x → param f y→ x=y)∧(∀ x y, 𝒩 x = 𝒩 y →  ≡^𝒩  x y).
```

Fig. 9. Valid numbering

class, and ordered w.r.t. reverse inclusion of equivalence kernels—recall that the equivalence kernel of $\mathcal{N}$ is the relation $\sim$ defined by $\texttt{x} \sim \texttt{y}$ if and only if $\mathcal{N}\ x = \mathcal{N}\ y$. Viewing the result of the analysis as a post-fixpoint is the key to our second component, a validator that checks whether a numbering $\mathcal{N}$ is indeed a post-fixpoint of the analysis on a program `p`, and if so returns an optimized SSA program `tp`. The validator is programmed in Coq, and is accompanied with a proof that optimized programs preserve the behaviors of the original programs.

The notion of valid numbering is formally defined in Fig. 9. First, we define for each numbering $\mathcal{N}$ the relation $\equiv^{\mathcal{N}}$ as the smallest reflexive relation identifying: (i) registers whose assignments share the same operator and corresponding arguments are equivalent w.r.t. $\mathcal{N}$ (predicate `same_number`); (ii) registers that are defined in the same ϕ-block with equivalent arguments.Then, for a numbering $\mathcal{N}$ to be valid (see `GVN_spec`), its equivalence kernel must not contain a pair of distinct function parameters and it must moreover be included in $\equiv^{\mathcal{N}}$. The latter ensures the intended post-fixpoint property.

The crux of the correctness proof of the GVN validator is the correctness lemma for a valid numbering: if $\mathcal{N}$ is a valid numbering for `f`, and `rs` is a register state that can be reached at node `pc`, and `x` and `y` are two registers whose definition strictly dominate `pc`, then $\mathcal{N}\ \texttt{x} = \mathcal{N}\ \texttt{y}$ entails that `rs` holds equal values for `x` and `y`:

```
Lemma valid_numbering_correct : ∀ prog s sp pc rs m,
    wf_ssa_program prog → GVN_spec 𝒩→
      reachable prog (State s f sp pc rs m) → gamma 𝒩 pc rs.
```

where `gamma` is defined by

```
Definition gamma (𝒩:reg → reg) (pc:node) (rs: regset) : Prop :=
  ∀ x y: reg, def_sdom f x pc → def_sdom f y pc →  𝒩 x = 𝒩 y → rs#x = rs#y.
```

and `def_sdom f x pc` states that the definition of `x` in `f` strictly dominates `pc`. Let us illustrate this property with Fig. 3; registers $\texttt{x}_2$ and $\texttt{y}_2$ share the same numbering; they are indeed equal just after the assignment of $\texttt{y}_2$ but not before.

Next, we describe the Coq implementation for optimizing SSA programs. The implementation takes as input a numbering $\mathcal{N}$, and a partial mapping `crep` that takes as input a register `x` and node `pc` and returns, if it exists, a register `y` such that `x` and `y` are related by the equivalence kernel of $\mathcal{N}$, and the definition of `y` strictly dominates `pc`. For efficiency reasons, we do not check the correctness of `crep` a priori, but lazily during the construction of the optimized program. The optimizer proceeds as follows: first, it checks whether $\mathcal{N}$ satisfies the predicate `GVN_spec`. Then, for each assignment (`Iop op args x pc`) of the original SSA program, the optimizer checks whether `crep` provides a canonical representative `y` for `x` at node `pc`. If so, it checks whether the definition of `y` strictly dominates `pc`; this is achieved by means of a dominance analysis, computed directly inside Coq with a standard dataflow framework *a la* Kildall. Provided `y` is validated, we can safely replace the previous instruction by a move from `y` to `x`.

We conclude by commenting briefly on the soundness proof of the transformation. It follows a standard forward simulation proof where the correctness of the numbering is proved at the same time as the simulation itself. Noticeably, the CFG normalization turned out to be extremely valuable for this proof. Indeed, consider a step from node `pc` to node `pc`$'$: we have to prove that (`gamma` $\mathcal{N}$ `pc`$'$ `rs`) holds, asumming (`gamma` $\mathcal{N}$ `pc rs`). We reason by case analysis: if the instruction at `pc` is not an `Inop` instruction, we know by normalization that `pc`$'$ is not a junction point. In this case, (`def_sdom f x pc`$'$) is equivalent to (`def_sdom f x pc`) $\vee$ (`def f x pc`) which is particularly useful to exploit the hypothesis that (`gamma` $\mathcal{N}$ `pc rs`) holds.

7 Implementation and experimental results

We have plugged in Compcert 1.8.2 our SSA middle-end made of (i) a Coq normalization (ii) an Ocaml SSA generator and its Coq validator; (iii) an Ocaml GVN inference tool and its Coq validator; (iv) a Coq de-SSA transformation. Our formal development adds 15.000 lines of Coq code and 1.000 lines of Ocaml to the 80.000 lines of Coq and 1.000 lines of Ocaml provided in CompCert. It does not add any axioms to CompCert. We use the Coq extraction mechanism to obtain a SSA-based certified compiler, that we evaluate experimentally using the CompCert benchmarks. These include around 75.000 lines of C code, and fall into three categories of programs (from 20 to 5.000 LoC): small computation kernels, a raytracer, and the theorem prover Spass[5]. Below we briefly comment on three key points: efficiency of the SSA validator; effectiveness of the GVN optimizer; efficiency of generated code.

SSA validator In order to be practical, validators must be more efficient than state-of-the-art implementations of the transformations that they validate. At first sight, this criterion may seem too demanding for SSA, since generation into SSA form is performed in almost linear time. However, experimental results are surprisingly good: overall converting a program into SSA form takes approximately twice longer than type-checking the output program. In more detail, the times for SSA generation—specialized to pruned SSA—distribute as follows:

[5] Spass is the largest (69.073 LoC), we only use it to evaluate the compilation time.

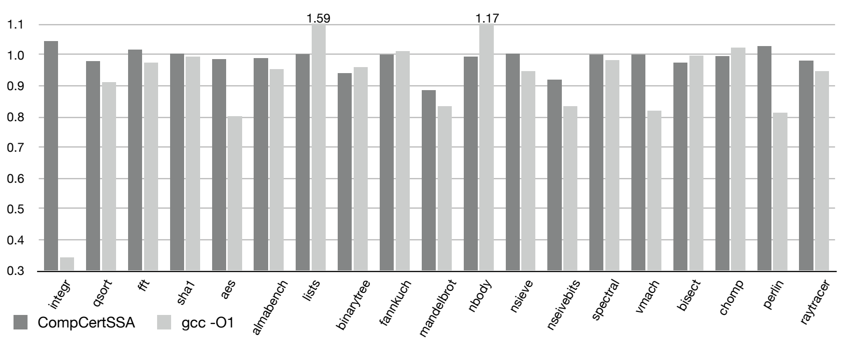


Fig. 10. Execution times of generated code

(i) 9% for normalization of RTL; (ii) 37% for liveness analysis of RTL (the liveness analysis is provided in the CompCert distribution); (iii) 35% for conversion to SSA using the untrusted OCaml implementation (based on state-of-the-art algorithms); (iv) 19% for validation using the verified validator. This distribution appears to be uniform on all benchmarks except on the biggest functions where the liveness analysis exhibits a non-linear complexity.

GVN optimizer We measure the effectiveness of our GVN analyzer by performing a GVN-based CSE right after (Local Value Numbering) LVN-based CSE implemented by CompCert, counting how many additional `Iop` instructions are optimized by this additional CSE phase. To keep the comparison fair, we allow CompCert CSE to optimize around function calls—this is disabled in CompCert to keep the register pressure low. The overall improvement is significative: our global CSE optimizes an additional 25% of `Iop`.

Generated code To assess the efficiency of the generated code, we have compiled the benchmarks with three compilers: CompCert, our version of CompCert extended with a SSA middle-end (CompCertSSA), and `gcc −O1`. Fig. 10 gives the execution times *relative* to Compcert (shorter bars mean faster) on PowerPC. The test suite is too small to draw definite conclusions, but the results are encouraging. Our version of CompCert performs slightly better than CompCert. We expect that performance improves significantly by enhancing our middle-end with additional optimizations, and by relying on an SSA-based register allocator.

8 Related Work

Machine-checked formalizations Blech *et al* [3] use the Isabelle/HOL proof assistant to verify the generation of machine code from a representation of SSA programs that relies on term graphs. While graph-based representations may be useful for the untrusted parts of our compiler, they increase the complexity of the formal SSA semantics, and make it a greater challenge to verify SSA-based optimizations. They do not provide an algorithm to convert into SSA form, and leave as future work proving the correctness of SSA-based optimizations. Mansky and Gunter [15] use Isabelle/HOL to formalize and verify the conversion of CFG programs into SSA form. However, their transformation may yield non-minimal SSA, and does not aim extraction into efficient code. Moreover, it is not clear whether their semantics of SSA can be used to reason about optimizations. Zhao *et al* [26] formalize the LLVM intermediate representation in Coq. They define and relate several formal semantics of LLVM, including a static and dynamic semantics. They show how simple code motions can be validated with a simulation relation based on symbolic evaluation, and plan to extend the method to other transformations such as dead code elimination or constant propagation. Finally, there are several machine-checked accounts of Continuation Passing Style translations, e.g. [8], closely related to conversion to SSA form [2].

Translation validation and type systems Menon *et al* [17] propose a type system that can be used to verify memory safety of programs in SSA form, but their system does not enforce the SSA property. Matsuno and Ohori [16] define a type system equivalent to SSA: every typable program is given a type annotation makeing explicit def-use relations. Their type system is similar to ours except they type check one program w.r.t. annotations while we type check a pair of a RTL and a SSA program. They show that common optimizations such as dead code elimination and CSE are type-preserving. But they do not prove the semantics preservation of the optimizations. Stepp *et al* [21] report on a translation validator for LLVM. Their validator uses Equality Saturation [22], which views optimizations as equality analyses. Their tool does not validate GVN. Tristan *et al* [23] independently report on an a translation validator for LLVM's interprocedural optimizations. This tool supports GVN, but is currently not certified.

9 Conclusion and Future Work

Our work shows that verified and realistic compilers can rely on a SSA-based middle-end that implements state-of-the-art algorithms, and opens the way for a new generation of verified compilers based on SSA. A priority for further work is to achieve a tighter integration of our middle-end into CompCert. There are three immediate objectives: (i) enhancing our SSA middle-end to handle memory aliases as done by CompCert RTL-based middle-end, (ii) implementing a SSA-based register allocator [9], and (iii) verifying more SSA-based optimizations, including lazy code motion [11]—we expect that our implementation of GVN will provide significant leverage there. Eventually, it should be possible to shift all CompCert optimisations into the SSA middle-end. In the longer term, it would be appealing to apply our methods to LLVM, building on [23, 21, 26].

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